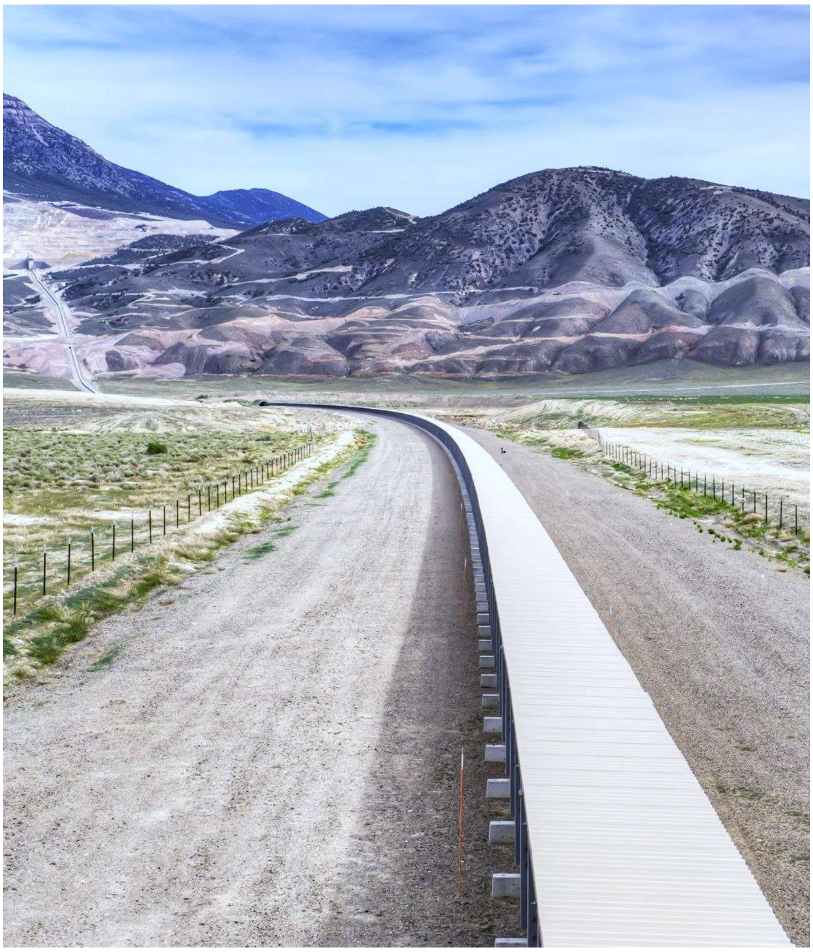

A THEORETICAL MODEL TO EVALUATE BELT WEAR EFFECTS ON LAGGING STRESS

RESEARCH TOPIC

This report summarizes the development of a theoretical model to evaluate the shear stresses experienced in conveyor pulley lagging due to belt cover wear.

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Introduction

Overland Conveyor Company was contracted by Elastotec Pty Ltd to develop a theoretical model to study the effects of conveyor belt cover wear on pulley lagging as a part of their Research and Development studies.

Concept of Study

Conveyor pulley lagging can experience wear and/or failure due to high stresses imposed upon it from the belt. One recent area of concern where field observations consistently show reduced expected lagging wear life is where lagging functions under high belt tension while also being in contact with the "dirty" side (or top cover) of the conveyor belting on non-driven conveyor pulleys.

Overland Conveyor Company has theorized that these high-tension bend pulley applications are problematic due to the potential likelihood that the conveyor belting cover has worn down along the center 1/3 of the belt, while the edges of the belt maintain their full cover thickness. This theory can be summarized within the following points:

- Existing belt cover wear causes some belt cords to travel around a pulley at a reduced radius and therefore shorter circumference compared to other cords on the belt [1]
- This will impose a combination of circumferential shear stress with potential slip at the lagging to belt interface (similar to a drive pulley application).
- This will also impose reduced radial belt pressure thus reducing available friction to prevent slip.
- These belt tension differences will impose a shear stress through the belt cover and lagging contact.

The methodology and Mathcad model has been built to evaluate the theory posed above. The primary goal of the model is to communicate the theoretical shear stress effects that belt wear may create within lagging on these high tension bend pulley applications. The evaluation process has the following steps.

1. Calculate the tension in each cord across the width of the belt. (MathCAD model)
    a. It should be noted that the tension in each cord will vary in two dimensions:
        i. Across the width of the belt
        ii. Throughout the contact with the pulley lagging and the belt (Nip Point to middle of belt wrap - 0 to 90 degrees of wrap)
    b. Compare Nip-Point Tensions to Mid-Wrap Tensions to determine worst case Tension differences (Tension differences would induce shear stress/strain in belt/lagging rubber).
2. Take these worst case tension differences and input them into OCC's Lagging Analyst software to evaluate rubber shear stress.

[1] Assumption: All cable length change is in the wear areas. In reality, some cables see higher tension/stretch causing lagging shear in the opposite direction for an averaging effect.

Please note that this work is suitable for comparison and understanding, but may not predict actual wear due to simplifying assumptions (discussed in some detail below).

Methodology and Assumptions

A cable load sharing model based on a paper by Hedgepeth (Hedgepeth, 1961) is used to predict the belt tensions and stretch across the belt width and from the center of pulley wrap to 'infinity'. (Many other papers are written on the prediction and effects of shear lag -the term for the effect of stiffening effect of adjacent material.) Hedgepeth, a planar model, was selected as an early work with inputs similar to steel cable belts. While Hedgepeth[2] considered the effect of broken cables, it allows a simpler input of individual cable length change across the belt width. Inputs to the model are cable tension modulus[3], cable diameter, cable spacing, and rubber modulus[4] as well as the cable length shortening. Sample results are provided below. Other operating conditions influence the effect of belt changes on shear/slip/wear.

The cable stretch and tension distributions provide a basis for the lagging shear strain similar to that in Lagging Analyst. The tension change from the center to end of pulley wrap is suggested as an index for future wear but more effects may be found with direct use of Lagging Analyst.

Fixed Parameters for this analysis are as follows:

- Pulley Wrap – 180 degrees
- Belt Wear on the cover for the center 1/3 of the belt (parabolic wear pattern)
- Belt Original Top Cover Thickness (18mm)
- Steel Cord Belt with a Width of 1800 mm

Variable Parameters for this analysis are as follows:

- Belt Tension:
  - 525 kN/m – ST3150 Belting – Cord Diameter 7.6 mm, Cord Pitch 15 mm
  - 750 kN/m – ST4500 Belting – Cord Diameter 9.3 mm, Cord Pitch 16 mm
  - 1050 kN/m – ST6300 Belting – Cord Diameter 12.3 mm, Cord Pitch 20 mm
  - 1250 kN/m – ST7500 Belting – Cord Diameter 13.2 mm, Cord Pitch 19 mm
  - Pulley Diameters were varied to match each belt rating's recommended high tension pulley diameter from readily available published data.
  - Cord Diameter, Pitch, and Elastic Modulus were grabbed from readily available published data from a large Conveyor Belt Manufacturer.
- Belt Cover Wear:
  - 5 mm, 10 mm, 15 mm

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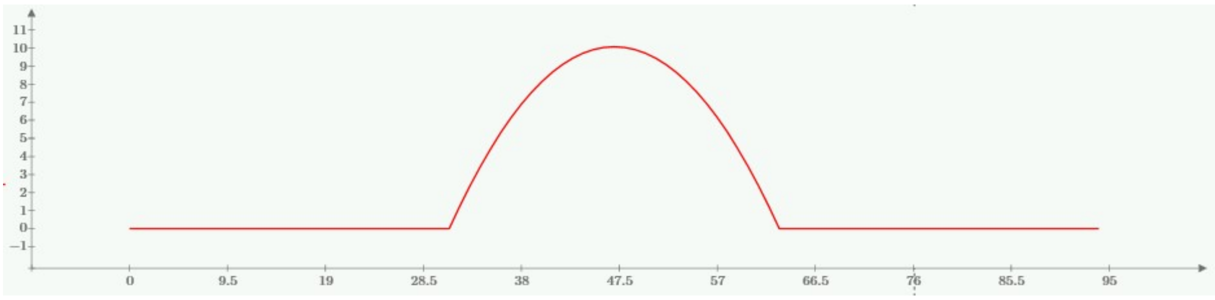
[2] Assumption: The current belt model assumes an infinitely wide belt. (<5% effect). It does not include any resistance due to lagging and cover stiffness.

[3] Assumption: Belt cords are considered to stretch similar to typical industry cables.

[4] Assumption: Belt shear longitudinal modulus G is selected from the 5741 Master Curve at the pulley rotation frequency. The rubber is guessed to be resisting shear differences between steel cables at a distance of up to two times the steel cord diameter.

Results and Basic Discussion

Within the MathCAD model, the analysis starts by calculating the relative radial steel cord positions within the belt across the width of the belt due to belt wear. The graph below shows the results of this calculation.



Plot: Relative Cord Position vs. Cord Number across the belt width

Evaluating this graph above, it can be clearly seen that a parabolic shift in steel cord position occurs across the center 1/3 of the belt. In this model, the cords in this center portion of the belt will be closer to the pulley lagging than the cords in the edge of the belt, with the cord at the absolute center of the belt being 5mm, 10mm, or 15mm closer to the lagging (depending on which results-case we are evaluating).

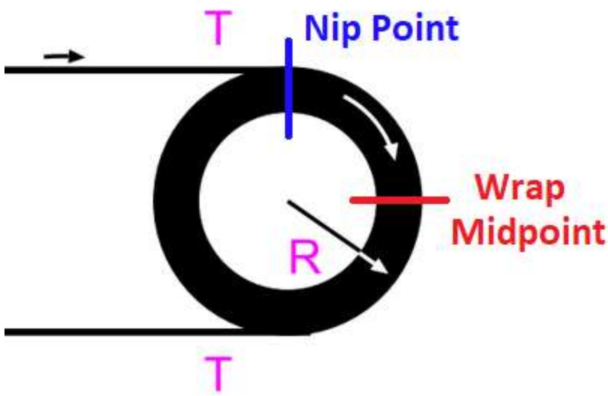
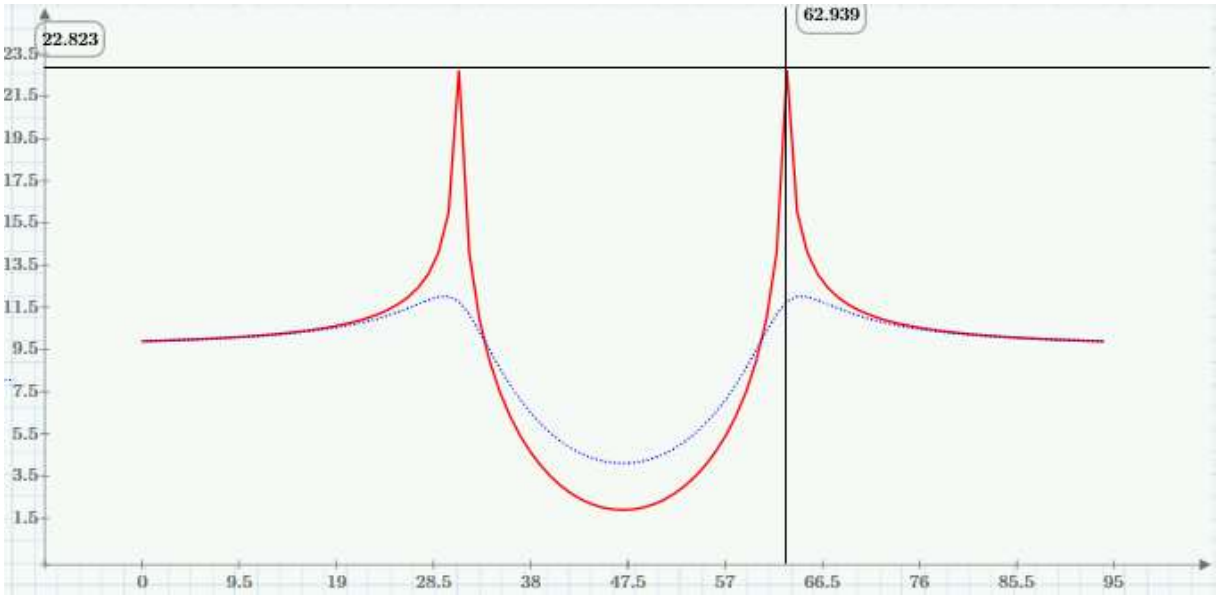


Diagram: Model setup showing nomenclature for Nip point and Wrap Point

The Hedgepeth model then allows us to calculate the theoretical steel cord cable stresses at the nip-point contact between belt and lagging, as well as at the middle of the wrap of the pulley. Differences in tension between nip point and wrap midpoint would create a shearing stress within the lagging and belt cover.



Plot: Tension (kN) within the steel cord cables at the nip point (blue) and wrap midpoint (red) vs Cord Number across the belt width

Evaluating the graph above, we can see that cable tension is reduced throughout the worn middle section of the belt, and the maximum cable tension occurs in the first cable in the non-worn section of the belt next to where the belt cover wear starts. The cables that see this maximum tension are taking on additional load to compensate for the reduced tension through the cables in the middle of the belt.

Furthermore, There can clearly be seen a difference in cable tensions at the nip point (blue line) when compared to the cable tension in the middle of the pulley wrap (red line). The tension difference on the cord with peak tension and the tension difference on the cord in the exact middle of the belt are used for further analysis within lagging analyst to determine the shear stress developed in the lagging. The following table summarizes these lagging analyst stress results:

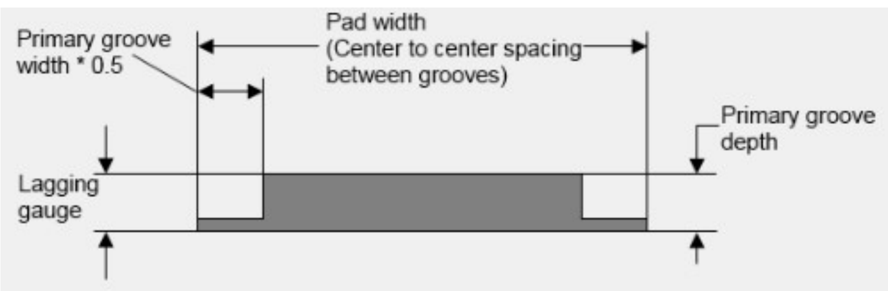
Results for 5 mm of Belt Cover Wear Over Center 1/3 of the Belt				
Belt Tension (kN)	Lagging Shear Stress (kPa)		Required Euler Friction to prevent Slip	
	Belt Mid Point	1/3 Away from Edge	Belt Mid Point	1/3 Away from Edge
2250	18.04	144.05	0.02	0.13
1688	18.04	144.05	0.03	0.17
1125	18.04	144.05	0.06	0.24
563	18.04	144.05	0.24	0.40

Results for 10 mm of Belt Cover Wear Over Center 1/3 of the Belt				
Belt Tension (kN)	Lagging Shear Stress (kPa)		Required Euler Friction to prevent Slip	
	Belt Mid Point	1/3 Away from Edge	Belt Mid Point	1/3 Away from Edge
2250	39.11	289.11	0.06	0.24
1688	39.11	289.11	0.10	0.30
1125	39.11	289.11	0.25	0.40
563	39.11	289.11	Infinity (Full Slip)	0.61

Results for 15 mm of Belt Cover Wear Over Center 1/3 of the Belt				
Belt Tension (kN)	Lagging Shear Stress (kPa)		Required Euler Friction to prevent Slip	
	Belt Mid Point	1/3 Away from Edge	Belt Mid Point	1/3 Away from Edge
2250	61.06	434.38	0.13	0.32
1688	61.06	434.38	0.26	0.40
1125	61.06	434.38	Infinity (Full Slip)	0.52
563	61.06	434.38	Infinity (Full Slip)	0.74 (slip)

Evaluating the results above:

- As expected, when belt wear is increased, shear stress on the lagging is increased.
- Belt Tension does not have an effect on shear stress, however, it does have a large effect on the normal pressure between the belt and lagging, which greatly effects the required euler friction to prevent slip.
- As the Belt tension drops in the 10mm and 15 mm wear models, the tension in the center of the belt turned negative. This would result in a complete loss of normal pressure between the belt and the lagging virtually ensuring full contact slip at the center of the belt. These results are reported with a “Infinity (Full Slip)” for the Required Euler Friction output.
- We know from our work with Dr. Bo Persson, that slip between surfaces is not fully quantified within the Euler friction model, and that slip is nearly always occurring, and the ultimate question is how much slip is occurring. However, for a simplified understanding of the model, Euler slip is used to evaluate this model.
- An ST7500 belt was used for the above calculation with a pulley diameter of 2500 mm (recommended high tension pulley diameter from catalogue).
- For the Lagging Analyst calculations, it was assumed that the lagging was 15 mm thick, 75 Shore A durometer, 150 mm pad width, groove width of 15 mm, and groove depth of 5mm (image below).



To understand the effect of Belt Construction on the results, the model was used to calculate the shear stress in the lagging for various belt constructions being operated under high tension (belt tension that creates a 6:1 safety factor for each belt), catalogue value for high tension pulleys for each belt, and 10 mm of belt cover wear. The results are as follows:

Results for 10 mm of Belt Cover Wear Over Center 1/3 of the Belt				
Belting Selection	Lagging Shear Stress (kPa)		Required Euler Friction to prevent Slip	
	Belt Mid Point	1/3 Away from Edge	Belt Mid Point	1/3 Away from Edge
ST7500	39.11	289.11	0.02	0.24
ST6300	43.51	341.05	0.12	0.28
ST4500	40.04	335.80	0.18	0.38
ST3150	38.27	329.47	0.48	0.49

Here we can see that the Belt manufacturer recommended pulley diameter allow for a relatively consistent shear stress result for each belt construction. 10 mm of wear creates ~40 kPa of shear stress in the lagging at the belt mid point, and ~330 kPa of shear stress in the lagging 1/3 of the way from the belt edge. The reduced tension in lower rated belts increases the risk of slip.

We should recognize that the cleanliness of the belt cover plays a very large roll in the expected available friction between the lagging and the belt cover. And any amount of dirt/dust/moisture may have the combined effect of reducing friction and increasing wear. Furthermore, slipping under higher normal pressures (higher belt tensions) will result in more wear than slipping under lower normal pressures (lower belt tensions). This model does not attempt to predict expected frictions or amount of wear resulting from the lagging shear stresses.

OCC believes the above results are fit for relative order of magnitude comparison of results to better understand how lagging stress is effected by belt cover wear.

Potential for Additional work

As stated above, the methodology and Mathcad model has been built to evaluate the theory posed at the beginning of the report. The primary goal of the model is to communicate the theoretical shear stress effects that belt wear may create within lagging on these high tension bend pulley applications. The results bear this in mind.

There is the potential for additional work to increase the confidence in the above results. These include the following:

- Consider +/- shear strain from +/- belt tension change as the real case.
- Use FEA to incorporate a better understanding of the effective rubber shear between cables.
- Perform Rubber Master Curve analysis of typical intercord rubber to get a better value for shear modulus.
- Incorporate the Dr. Persson model for mu-slip friction into the analysis to better predict how much slip may occur.
- Upgrade the Hedgepeth model for finite belt width allowing extending this model to other influences (transitions, curves) and purposes.
- Upgrade Hedgepeth model to include lagging and cover shear stiffness.

References

Hedgepeth, J. M. (1961). *Stress Concentrations in Filamentary Structures.* Washington: National Aeronautics and Space Administration.

Mathematical Model For Belt Cover Center Wear on Lagging Stresses
Created by Al Reicks for Overland Conveyor Company
May 2020

HedgePeth 1961 Model for Cable Load Sharing:
nondimensional form

force	displacement	length
$p_n = p \cdot P_n$	$u_n = p \cdot \sqrt{\dfrac{d}{EA \cdot G \cdot h}} \cdot U_n$	$x = \sqrt{\dfrac{EA_c \cdot d}{G \cdot h}} \cdot \xi$

Inputs to the Model:

$T_{rating} := 525 \dfrac{kN}{m}$	Belt Tension Rating
$dia := 7.6\ mm$	Steel cord diameter for belt
$pitch := 15\ mm$	Steel cord pitch (distance between cord centers)
$BeltModulus := 220000 \dfrac{kN}{m}$	Belt Elastic Modulus
$D_p := 1600\ mm$	Pulley Diameter
$wrap := 180\ deg$	Belt Wrap around pulley
$BW := 1800\ mm$	Belt Width
$T := 525 \dfrac{kN}{m} \cdot BW$	Operating Belt Tension
$G_r := 8 \cdot 10^6\ Pa$	Shear Stiffness of the Inter-Cord Rubber at Operating Velocity
$G_{stretch} := .0000128$	ref loosco stretch factor
$wear_{max} := -10\ mm$	Amount of Belt Cover Wear

Calculations

$n_a := \text{floor}\left(\dfrac{BW}{pitch}\right) = 120$ Number of Cords in the belt

$$E_{Eff} := \frac{400\ lbf}{G_{stretch} \cdot \pi \cdot in^2} = \left(6.858 \cdot 10^{10}\right)\ Pa \qquad E_{Eff} := \frac{BeltModulus \cdot BW}{n_a \cdot \pi \cdot \left(\dfrac{dia}{2}\right)^2} = \left(7.274 \cdot 10^{10}\right)\ Pa$$

$$EA := E_{Eff} \cdot \frac{\pi \cdot dia^2}{4} = \left(3.3 \cdot 10^6\right)\ N$$

$$G_h := G_r \cdot (pitch - dia) \cdot 2$$

$$scale_p := \frac{T}{n_a} \qquad scale_u := \frac{T}{n_a} \cdot \sqrt{\frac{BW}{n_a \cdot EA \cdot G_h}} \qquad scale_x := \sqrt{\frac{EA \cdot BW}{n_a \cdot G_h}}$$

$$scale_p = \left(7.875 \cdot 10^3\right)\ N \qquad scale_u = 0.002\ m \qquad scale_x = 0.647\ m$$

$r := \text{floor}\left(\frac{n_a}{3}\right) = 40$ worn cables for center 1/3rd

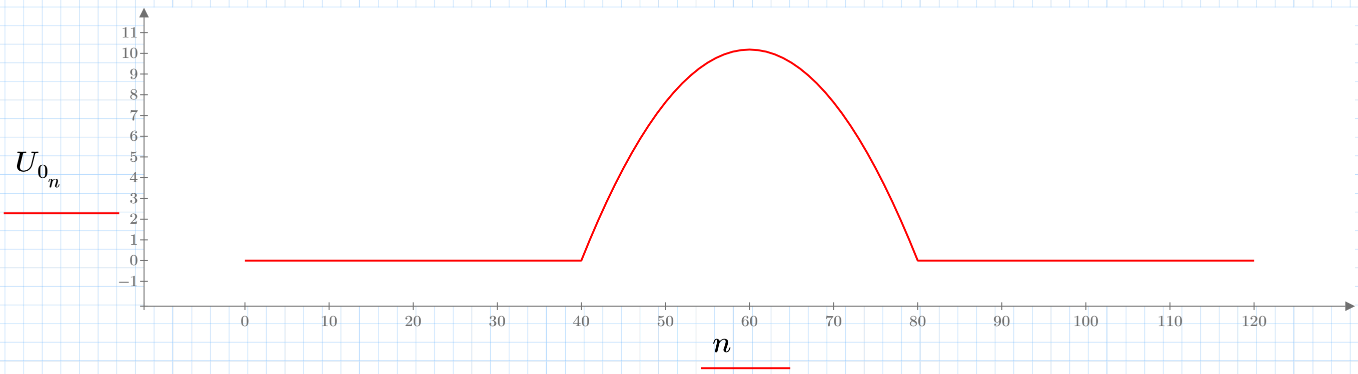
$n := 0, 1 .. n_a$ $U_{0_n} := 0$ $n_0 := \frac{n_a}{2} - \frac{r}{2}$ $n_0 = 40$ $n_r := n_0, n_0 + 1 .. n_0 + r$

Apply wear as a parabolic distribution

$$d_{wmx} := \frac{wear_{max}}{scale_u} \qquad d_n := 0\ m \qquad d_{n_r} := wear_{max} \cdot \left(1 - \left(\frac{\frac{n_a}{2} - n_r}{\frac{r}{2}}\right)^2\right)$$

Plot: Cord Relative Position vs Cord Number in Belt

$$U_{0_{n_r}} := -d_{n_r} \cdot \frac{wrap}{2 \cdot scale_u} \qquad U_{10_{n_r - n_0}} := U_{0_{n_r}}$$



Calculate Positions and Stresses for every cord

$\xi_{mx} := 15$ $\xi := 0, .05 \cdot \xi_{mx} .. \xi_{mx}$

$$U_1(\xi, n) := \xi + \sum_{m=0}^{r-1} \left(\frac{U_{10_m}}{\pi} \cdot \int_0^{\pi} \cos((n-m) \cdot \theta) \cdot e^{-2 \cdot \xi \cdot \sin\left(\frac{\theta}{2}\right)} \, d\theta\right)$$

$$U(\xi, n) := U_1(\xi, n - n_0)$$

$$U_{str}(\xi, n) := \sum_{m=0}^{r-1} \left(\frac{U_{10_m}}{\pi} \cdot \int_0^{\pi} \cos((n-m) \cdot \theta) \cdot e^{-2 \cdot \xi \cdot \sin\left(\frac{\theta}{2}\right)} \, d\theta\right)$$

$$U_2(\xi, n) := U_{str}(\xi, n - n_0)$$

$$P_1(\xi, n) := 1 + \sum_{m=0}^{r-1} \left(U_{10_m} \cdot \frac{-2}{\pi} \cdot \int_0^{\pi} \sin\left(\frac{1}{2} \cdot \theta\right) \cdot \cos((n-m) \cdot \theta) \cdot e^{-2 \cdot \xi \cdot \sin\left(\frac{\theta}{2}\right)} \, d\theta\right)$$

$$P(\xi, n) := P_1(\xi, n - n_0)$$

Major Results:

Cord Relative Position at different distances from the Pulley

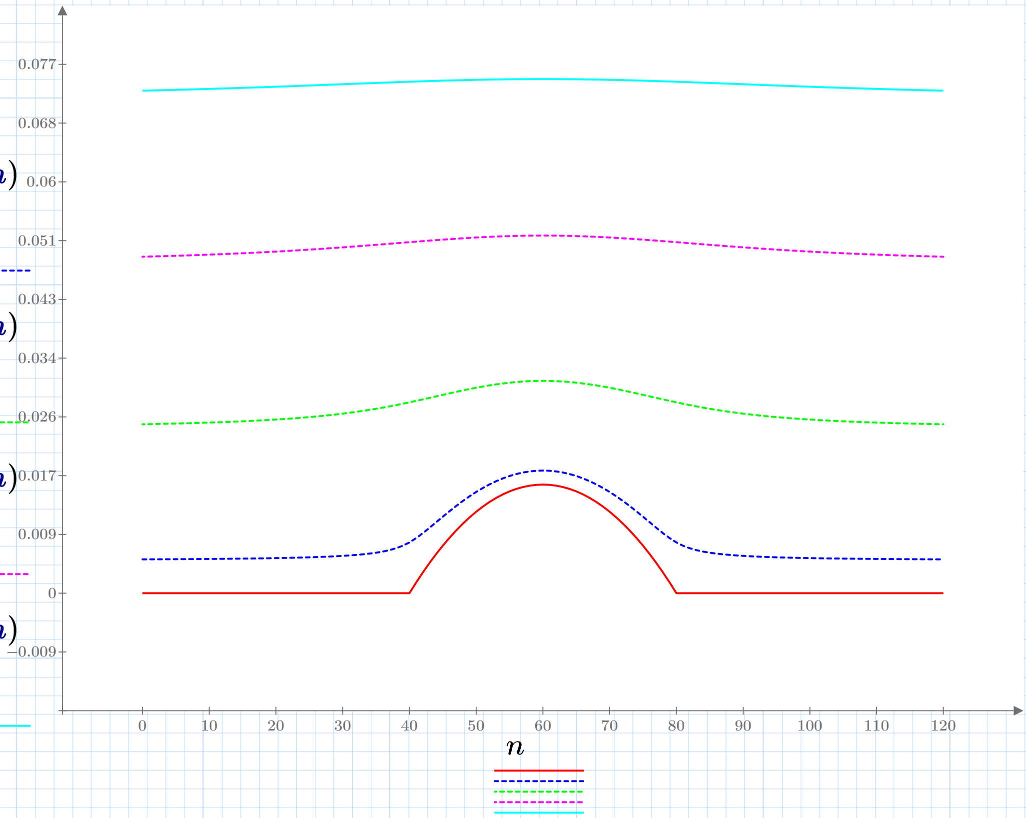
$U(0,n) \cdot scale_u \ (m)$

$U\left(\frac{2}{\frac{scale_x}{m}}, n\right) \cdot scale_u \ (m)$

$U\left(\frac{10}{\frac{scale_x}{m}}, n\right) \cdot scale_u \ (m)$

$U\left(\frac{20}{\frac{scale_x}{m}}, n\right) \cdot scale_u \ (m)$

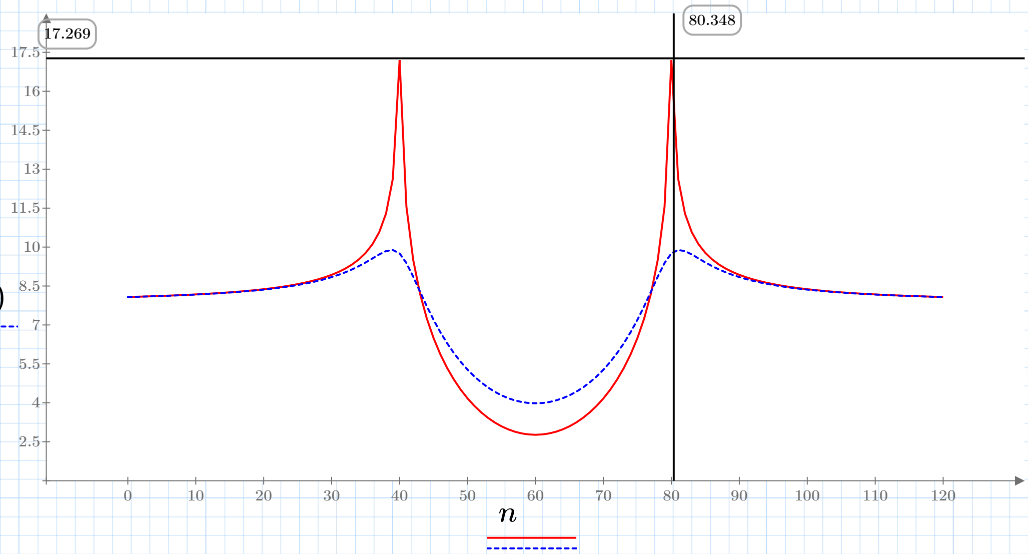
$U\left(\frac{30}{\frac{scale_x}{m}}, n\right) \cdot scale_u \ (m)$



Cord Tension at Nip Point and Middle of Pulley Wrap vs Belt Width

$P(0,n) \cdot scale_p \ (kN)$

$P(3.385,n) \cdot scale_p \ (kN)$



Cord Tensions at Middle of Belt

$P(3.385,60) \cdot scale_p = 3.983 \ kN$

$P(0,60) \cdot scale_p = 2.771 \ kN$

Cord Tensions at Peak Stress

$P(3.385,80) \cdot scale_p = 9.756 \ kN$

$P(0,80) \cdot scale_p = 17.195 \ kN$